VALSCALC

Chapter 9 Free-Response Practice Test

Directions: This practice test features free-response questions based on the content in Chapter 9: Parametric Equations and Polar Coordinates.

- **9.1**: Parametric Equations
- **9.2**: Differentiating and Integrating Parametric Functions
- **9.3**: Polar Coordinates and Functions
- **9.4**: Differentiating Polar Functions
- **9.5**: Areas with Polar Curves
- **9.6**: Additional Calculus with Parametric and Polar

For each question, show your work. If you encounter difficulties with a question, then move on and return to it later. Follow these guidelines:

- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit of 90 minutes.
- After you complete all the questions, score yourself according to the Solutions document. Note any topics that require revision.

Visit valscalc.com to access a free online calculus textbook and additional multiple-choice and free-response practice exams.

Parametric Equations and Polar Coordinates

Number of Questions—10

Suggested Time—1 hour 30 minutes

NO CALCULATOR

Scoring Chart

Section	Points Earned	Points Available
Short Questions		25
Question 6		15
Question 7		15
Question 8		15
Question 9		15
Question 10		15
TOTAL		**100**

Short Questions

1. The curve C is parameterized by the equations $x = \ln t$ and $y = \tan^{-1} t$ for $t > 0$. Write an equation of the line tangent to the curve C when $t = 1$. *(5 pts.)*

2. Determine the area enclosed by the lemniscate $r^2 = 16 \sin 2\theta$. *(5 pts.)*

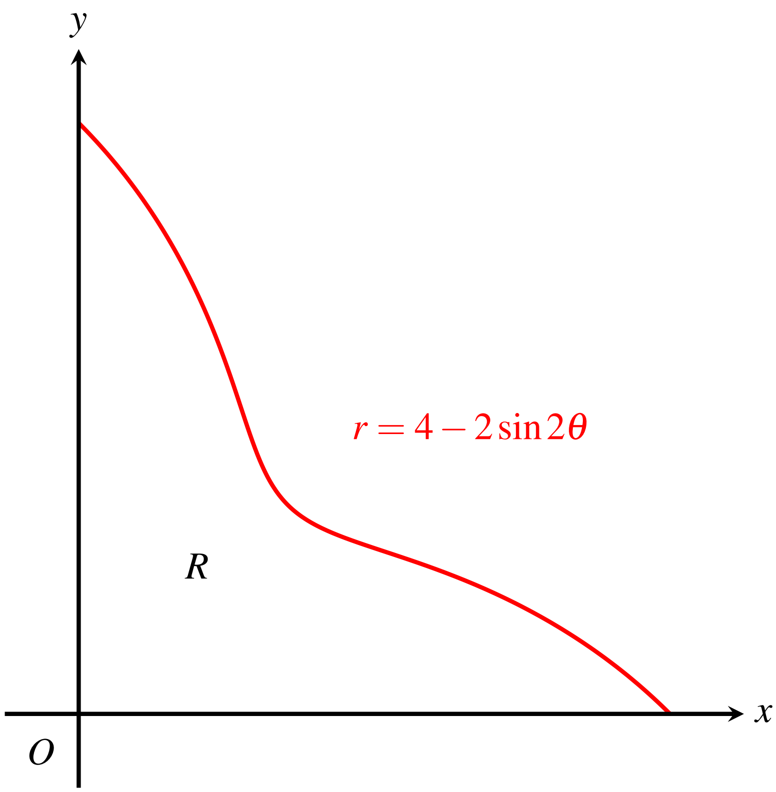
3. A particle travels clockwise along a circle of radius 3 centered at $(4,-2)$. If the particle begins at $(7,-2)$ and finishes one complete revolution on $0 \leqslant t \leqslant \pi$, then write parametric equations to model the particle's movement. *(5 pts.)*

4. Calculate the surface area of revolution in rotating the curve parameterized by $x = -5\cos t$ and $y = 2 + 5\sin t$, $0 \leqslant t \leqslant \dfrac{\pi}{2}$, about the x-axis. *(5 pts.)*

5. Calculate the area of the region that is inside the limacon $r = 2 + \cos\theta$ and also inside the circle $r = 2$. *(5 pts.)*

Long Questions

6. The graph of the polar function $r = 4 - 2\sin 2\theta$ for $0 \leqslant \theta \leqslant \dfrac{\pi}{2}$ is shown in the following figure. Let R be the region bounded by the polar curve in the first quadrant.



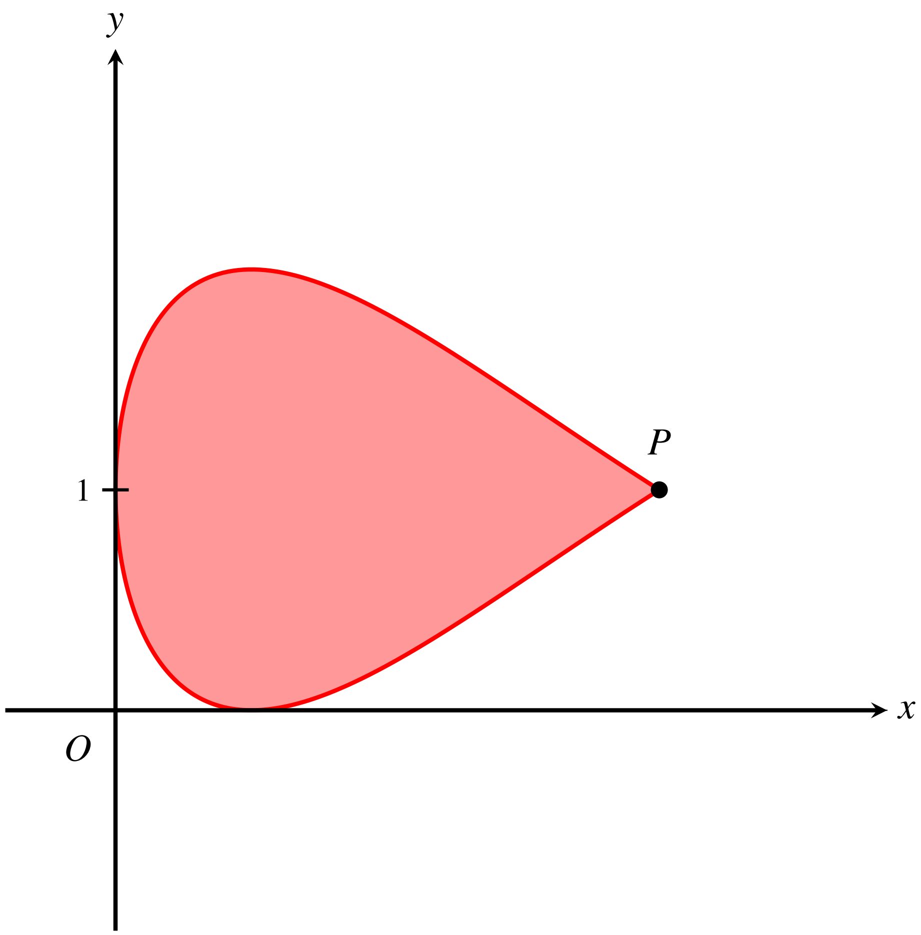
(a) Calculate the values of r and $\dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ at $\theta = \dfrac{\pi}{6}$. At $\theta = \dfrac{\pi}{6}$, is the polar curve moving toward or away from the origin? (3 pts.)

(b) Calculate the area of R. (5 pts.)

(c) Write, but do not evaluate, an integral expression whose value gives the arc length of the polar curve from $\theta = 0$ to $\theta = \dfrac{\pi}{2}$. (3 pts.)

(d) Set up, but do not evaluate, an integral expression whose value gives the surface area of revolution upon revolving $r = 4 - 2\sin 2\theta$ around the y-axis. (4 pts.)

7. A teardrop is parameterized by the functions $x = \dfrac{t^2}{4}$ and $y = 1 + \sin t$ for $-\pi \leqslant t \leqslant \pi$, as shown in the following figure.



(a) Given that the y-coordinate of point P is 1, find the x-coordinate of P. *(2 pts.)*

(b) Using differentiation, find the value of t at which the graph has a vertical tangent. *(4 pts.)*

(c) Calculate the area of the teardrop. *(5 pts.)*

(d) Write, but do not evaluate, an integral whose value gives the arc length of the teardrop. *(4 pts.)*

8. Consider the curve parameterized by $x = 9 + 4t - \dfrac{t^2}{2}$ and $y = \sqrt{t^2 + 1}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{t}{(4-t)\sqrt{t^2+1}}$. *(4 pts.)*

(b) Find all the values of t at which the curve has horizontal tangents and vertical tangents. *(3 pts.)*

(c) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(5 pts.)*

(d) Determine the values of t at which the curve has inflection points. *(3 pts.)*

9. Consider the cardioid defined by the polar function $r = 3 - 3\cos\theta$ for $0 \leqslant \theta \leqslant 2\pi$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\cos\theta - \cos 2\theta}{-\sin\theta + \sin 2\theta}$. *(5 pts.)*

(b) Write an equation of the line tangent to the graph of $r = 3 - 3\cos\theta$ when $\theta = \dfrac{\pi}{2}$. *(3 pts.)*

(c) Find all the values of θ in $[0, 2\pi]$ at which the cardioid has horizontal tangents. *(4 pts.)*

(d) Determine all the values of θ in $[0, 2\pi]$ at which the cardioid has vertical tangents. *(3 pts.)*

10. A projectile is launched upward from the ground with an initial speed of 40 meters per second at an angle of 45° above the horizontal.

(a) Write parametric equations to model the projectile's motion in the horizontal and vertical directions as functions of time t in seconds. *(4 pts.)*

(b) Calculate the projectile's height after 1 second. *(2 pts.)*

(c) Find the exact time at which the projectile reaches the ground. How far does the projectile travel? *(5 pts.)*

(d) Set up, but do not evaluate, an integral that equals the length of the projectile's trajectory. *(4 pts.)*

This marks the end of the test. The solutions and scoring rubric begin on the next page.

Short Questions (5 points each)

1. Differentiating both functions gives

$$\frac{dx}{dt} = \frac{1}{t}, \quad *$$

$$\frac{dy}{dt} = \frac{1}{t^2+1}. \quad *$$

Then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1/(t^2+1)}{1/t} = \frac{t}{t^2+1}. \quad *$$

At $t = 1$,

$$x = \ln 1 = 0 \qquad \text{and} \qquad y = \tan^{-1} 1 = \frac{\pi}{4}. \quad *$$

In addition,

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{1}{(1)^2+1} = \frac{1}{2}.$$

Accordingly, an equation of the tangent line at $t = 1$ is

$$y - \frac{\pi}{4} = \tfrac{1}{2}(x-0) \qquad \text{or} \qquad \boxed{y = \frac{x}{2} + \frac{\pi}{4}} \quad *$$

2. Note that $r = 0$ when $\theta = 0$ and $\theta = \dfrac{\pi}{2}$, so one loop of the lemniscate is traced out over $0 \leqslant \theta \leqslant \dfrac{\pi}{2}$. By symmetry, the lemniscate's area is twice the area of one loop—namely,

$$\begin{aligned} A &= 2 \cdot \tfrac{1}{2} \int_0^{\pi/2} r^2 \, d\theta \\ &= 2 \cdot \tfrac{1}{2} \int_0^{\pi/2} 16\,|\sin 2\theta| \, d\theta \\ &= 16 \int_0^{\pi/2} \sin 2\theta \, d\theta. \quad ** \end{aligned}$$

The absolute value bars are dropped because $\sin 2\theta$ is nonnegative on $\left[0, \frac{\pi}{2}\right]$. Then

$$A = -8\cos 2\theta \Big|_0^{\pi/2} \quad *$$

$$= \boxed{16} \quad **$$

3. The circle is centered at $(4, -2)$ and has radius 3, and the particle begins at $(7, -2)$. Therefore, the *preliminary* parametric equations are

$$x = 4 \pm 3\cos\omega t\,, \quad *$$

$$y = -2 \pm 3\sin\omega t\,, \quad *$$

where ω is a constant. The period is π, so

$$\omega = \frac{2\pi}{\pi} = 2\,. \quad *$$

In addition, the particle begins at the far-right of the circle and travels clockwise, so y must be decreasing at $t = 0$. Hence, we use

$$\boxed{x = 4 + 3\cos 2t} \quad *$$

$$\boxed{y = -2 - 3\sin 2t} \quad *$$

4. We find

$$\frac{dx}{dt} = 5\sin t \qquad \text{and} \qquad \frac{dy}{dt} = 5\cos t\,. \quad *$$

Thus, the surface area of revolution is

$$\begin{aligned} S &= \int_0^{\pi/2} 2\pi(2+5\sin t)\sqrt{(5\sin t)^2+(5\cos t)^2}\,\mathrm{d}t && * \\ &= 2\pi\int_0^{\pi/2}(2+5\sin t)\sqrt{25(\sin^2 t+\cos^2 t)}\,\mathrm{d}t \\ &= 10\pi\int_0^{\pi/2}(2+5\sin t)\,\mathrm{d}t && * \\ &= 10\pi(2t-5\cos t)\Big|_0^{\pi/2} && * \\ &= \boxed{10\pi^2+50\pi} && * \end{aligned}$$

5. From $\theta = -\dfrac{\pi}{2}$ to $\theta = \dfrac{\pi}{2}$, the region is a semicircle of radius 2, whose area is $\frac{1}{2}\pi(2)^2 = 2\pi$. But from $\theta = \dfrac{\pi}{2}$ to $\theta = \dfrac{3\pi}{2}$, the region is bounded by the limacon $r = 2+\cos\theta$. Hence, the area is

$$A = 2\pi + \tfrac{1}{2}\int_{\pi/2}^{3\pi/2}(2+\cos\theta)^2\,\mathrm{d}\theta\,. \qquad *$$

To simplify calculations, using symmetry gives

$$\begin{aligned} A &= 2\pi + 2\cdot\tfrac{1}{2}\int_{\pi/2}^{\pi}(2+\cos\theta)^2\,\mathrm{d}\theta \\ &= 2\pi + \int_{\pi/2}^{\pi}(4+4\cos\theta+\cos^2\theta)\,\mathrm{d}\theta \\ &= 2\pi + \int_{\pi/2}^{\pi}\left(4+4\cos\theta+\tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta\right)\mathrm{d}\theta \\ &= 2\pi + \int_{\pi/2}^{\pi}\left(\tfrac{9}{2}+4\cos\theta+\tfrac{1}{2}\cos 2\theta\right)\mathrm{d}\theta && * \\ &= 2\pi + \left(\tfrac{9}{2}\theta+4\sin\theta+\tfrac{1}{4}\sin 2\theta\right)\Big|_{\pi/2}^{\pi} && * \\ &= 2\pi + \left(\frac{9\pi}{4}-4\right) = \boxed{\frac{17\pi}{4}-4} && ** \end{aligned}$$

Long Questions (15 points each)

6. (a) We have

$$r\left(\frac{\pi}{6}\right) = 4 - 2\sin\left(\frac{\pi}{3}\right) = \boxed{4-\sqrt{3}}$$ *

In addition, $\frac{dr}{d\theta} = -4\cos 2\theta$, so

$$\left.\frac{dr}{d\theta}\right|_{\theta=\pi/6} = -4\cos\left(\frac{\pi}{3}\right) = \boxed{-2}$$ *

Because $r > 0$ and $\frac{dr}{d\theta} < 0$ (opposite signs), at $\theta = \frac{\pi}{6}$ the graph is moving

$$\boxed{\text{toward the origin}}$$ *

(b) The area of region R is

$$A = \tfrac{1}{2}\int_0^{\pi/2} (4 - 2\sin 2\theta)^2 \, d\theta$$ *

$$= \tfrac{1}{2}\int_0^{\pi/2} (16 - 16\sin 2\theta + 4\sin^2 2\theta) \, d\theta$$

$$= \tfrac{1}{2}\int_0^{\pi/2} (16 - 16\sin 2\theta + 2 - 2\cos 4\theta) \, d\theta$$ *

$$= \int_0^{\pi/2} (9 - 8\sin 2\theta - \cos 4\theta) \, d\theta$$

$$= \left(9\theta + 4\cos 2\theta - \tfrac{1}{4}\sin 4\theta\right)\Big|_0^{\pi/2}$$ *

$$= \boxed{\frac{9\pi}{2} - 8}$$ **

(c) With $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -4\cos 2\theta$, the arc length is

$$L = \int_0^{\pi/2} \sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$$ *

$$= \boxed{\int_0^{\pi/2} \sqrt{(4-2\sin 2\theta)^2 + (-4\cos 2\theta)^2}\,\mathrm{d}\theta}$$ **

(d) The curve lies a distance $x = r\cos\theta$ away from the y-axis, so the surface area of revolution is

$$S = \int_0^{\pi/2} 2\pi r\cos\theta \sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$$ *

$$= \boxed{2\pi \int_0^{\pi/2} (4-2\sin 2\theta)\cos\theta \sqrt{(4-2\sin 2\theta)^2 + (-4\cos 2\theta)^2}\,\mathrm{d}\theta}$$ ***

7. (a) Because $y = 1 + \sin t$, $-\pi \leqslant t \leqslant \pi$, we solve for t such that $y = 1$:

$$1 + \sin t = 1 \implies \sin t = 0 \implies t = -\pi, 0, \pi.$$ *

We evaluate $x = \dfrac{t^2}{4}$ at each t value:

$$x(-\pi) = \frac{(-\pi)^2}{4} = \frac{\pi^2}{4},$$

$$x(0) = \frac{0^2}{4} = 0,$$

$$x(\pi) = \frac{\pi^2}{4} = \frac{\pi^2}{4}.$$

Clearly, point P is not on the y-axis, so $x = 0$ is *incorrect*. Instead, the x-coordinate of P is

$$x = \boxed{\frac{\pi^2}{4}}$$ *

which is reached at both $t = -\pi$ and $t = \pi$.

(b) We have

$$\frac{dx}{dt} = \frac{t}{2} \qquad \text{and} \qquad \frac{dy}{dt} = \cos t\,.$$ *

Thus,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{t/2} = \frac{2\cos t}{t}\,.$$ *

A vertical tangent occurs when the denominator is 0 and the numerator is nonzero. The denominator equals 0 when $t = 0$; at the same time, the numerator is nonzero. Thus, a vertical tangent occurs when

$$t = \boxed{0}$$ **

(c) In Cartesian form, the area of the teardrop's top half is given by

$$A_{\text{top}} = \int_{x=0}^{x=\pi^2/4} (y-1)\, dx\,,$$

where y is the function representing the upper boundary of the teardrop. By symmetry, the entire teardrop's area is

$$A = 2A_{\text{top}} = 2\int_{x=0}^{x=\pi^2/4} (y-1)\, dx\,.$$ *

With $x = \frac{t^2}{4}$, the differential is $dx = \frac{t}{2}\,dt$. In addition, when $x = 0$, $t = 0$; when $x = \frac{\pi^2}{4}$, $t = \pi$ (for the upper half). Thus, the Substitution Rule gives

$$\begin{aligned} A &= 2\int_0^{\pi} [(1+\sin t) - 1]\left(\frac{t}{2}\right)\, dt \qquad ** \\ &= \int_0^{\pi} t\sin t\, dt\,. \end{aligned}$$

Using Integration by Parts, we attain

$$\begin{aligned} A &= (-t\cos t + \sin t)\Big|_0^{\pi} \qquad * \\ &= \boxed{\pi} \qquad * \end{aligned}$$

(d) With $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{t}{2}$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = \cos t$, the arc length is

$$L = \int_{-\pi}^{\pi} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$$ *

$$= \boxed{\int_{-\pi}^{\pi} \sqrt{\left(\frac{t}{2}\right)^2 + \cos^2 t}\,\mathrm{d}t}$$ ***

8. (a) Differentiating both parametric functions shows

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 4 - t\,,$$ *

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{t}{\sqrt{t^2+1}}\,.$$ *

Thus,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t}$$ *

$$= \frac{t/\sqrt{t^2+1}}{4-t}$$

$$= \boxed{\frac{t}{(4-t)\sqrt{t^2+1}}}$$ *

(b) The curve has horizontal tangents when the numerator is 0 and the denominator is nonzero. The only value satisfying both criteria is

$$t = \boxed{0}$$ *

Conversely, the curve has vertical tangents when the denominator is 0 and the numerator is nonzero. The denominator equals 0 when $t = 4$; at this value, the numerator is nonzero. Hence, the only vertical tangent is

$$t = \boxed{4}$$ **

(c) Differentiating the function in part (a), we see

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) &= \frac{(4-t)\sqrt{t^2+1}+t\sqrt{t^2+1}-\dfrac{t^2(4-t)}{\sqrt{t^2+1}}}{(4-t)^2(t^2+1)} \\
&= \frac{(4-t)(t^2+1)+t(t^2+1)-t^2(4-t)}{(4-t)^2(t^2+1)^{3/2}} \\
&= \frac{t^3+4}{(4-t)^2(t^2+1)^{3/2}} && {**}
\end{aligned}$$

Now, noting that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 4-t$,

$$\begin{aligned}
\frac{\mathrm{d}^2y}{\mathrm{d}x^2} &= \frac{\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)}{\mathrm{d}x/\mathrm{d}t} && * \\
&= \boxed{\frac{t^3+4}{(4-t)^3(t^2+1)^{3/2}}} && *
\end{aligned}$$

(d) Considering $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$, we have

$$\begin{aligned}
\frac{t^3+4}{(4-t)^3(t^2+1)^{3/2}} &= 0 && * \\
t^3+4 &= 0 \\
\implies t &= -\sqrt[3]{4}.
\end{aligned}$$

The sign of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ changes sign from negative to positive at this value, so the only inflection point occurs when

$$t = \boxed{-\sqrt[3]{4}} \qquad {**}$$

9. (a) (Throughout this problem, the double-angle identity for sine, $\sin 2x = 2\sin x\cos x$, is used to simplify calculations.) Because $x = r\cos\theta$ and $y = r\sin\theta$, we have

$$x = 3\cos\theta - 3\cos^2\theta\,,$$

$$y = 3\sin\theta - 3\cos\theta\sin\theta = 3\sin\theta - \tfrac{3}{2}\sin 2\theta\,.$$ *

Then

$$\frac{dx}{d\theta} = -3\sin\theta + 6\cos\theta\sin\theta = -3\sin\theta + 3\sin 2\theta\,,$$ *

$$\frac{dy}{d\theta} = 3\cos\theta - 3\cos 2\theta\,.$$ *

Now

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$ *

$$= \frac{3\cos\theta - 3\cos 2\theta}{-3\sin\theta + 3\sin 2\theta}$$

$$= \boxed{\frac{\cos\theta - \cos 2\theta}{-\sin\theta + \sin 2\theta}}$$ *

(b) At $\theta = \dfrac{\pi}{2}$, we have, from the definitions of x and y in part (a),

$$x = 0 \qquad \text{and} \qquad y = 3\,.$$ *

In addition, from the derivative function,

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = -1\,.$$ *

Thus, an equation of the tangent line at $\theta = \dfrac{\pi}{2}$ is

$$y - 3 = -1(x - 0) \qquad \text{or} \qquad \boxed{y = 3 - x}$$ *

(c) The graph has a horizontal tangent when the numerator is 0 and the denominator is nonzero.

Equating the numerator to 0, we attain

$$\begin{aligned}\cos\theta-\cos 2\theta&=0 \quad * \\ \cos\theta-(\cos^2\theta-\sin^2\theta)&=0 \\ \cos\theta-(\cos^2\theta-[1-\cos^2\theta])&=0 \\ 2\cos^2\theta-\cos\theta-1&=0\,.\end{aligned}$$

This equation is quadratic in $\cos\theta$, so factoring it yields

$$(\cos\theta-1)(2\cos\theta+1)=0\,, \quad *$$

which is satisfied by either $\cos\theta=1$ or $\cos\theta=-\frac{1}{2}$. In $[0,2\pi]$, the solutions are

$$\theta=0\,,\frac{2\pi}{3}\,,\frac{4\pi}{3}\,,2\pi\,. \quad *$$

However, the denominator equals 0 only at $\theta=0$ and $\theta=2\pi$, so horizontal tangents occur when

$$\theta=\boxed{\frac{2\pi}{3}}\,,\boxed{\frac{4\pi}{3}} \quad *$$

(d) The graph has a vertical tangent when the denominator is 0 and the numerator is nonzero. Equating the denominator to 0 shows

$$\begin{aligned}\sin\theta&=\sin 2\theta \quad * \\ \sin\theta&=2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta-\sin\theta&=0 \\ \sin\theta(2\cos\theta-1)&=0\,,\end{aligned}$$

which is satisfied by either $\sin\theta=0$ or $\cos\theta=\frac{1}{2}$. In $[0,2\pi]$, the solutions are

$$\theta=0\,,\frac{\pi}{3}\,,\pi\,,\frac{5\pi}{3}\,,2\pi\,. \quad *$$

The numerator is nonzero only for

$$\theta = \boxed{\frac{\pi}{3}}, \boxed{\pi}, \boxed{\frac{5\pi}{3}}$$ *

so the graph has vertical tangents at these values.

10. (a) The initial speed is $v_0 = 40$ m/sec, the initial height is $y_0 = 0$, the launch angle is $\alpha = 45^\circ$, and the acceleration due to gravity is $g = 9.8$ m/sec^2. Taking the upward direction to be positive, we write

$$\begin{aligned} x(t) &= v_0(\cos\alpha)t \\ &= 40(\cos 45^\circ)t \\ &= \boxed{20\sqrt{2}t} \end{aligned}$$ * *

and

$$\begin{aligned} y(t) &= y_0 + v_0(\sin\alpha)t - \tfrac{1}{2}gt^2 \\ &= 40(\sin 45^\circ)t - \tfrac{1}{2}(9.8)t^2 \\ &= \boxed{20\sqrt{2}t - 4.9t^2} \end{aligned}$$ * *

(b) The height after 1 sec, in meters, is

$$y(1) = \boxed{20\sqrt{2} - 4.9}$$ **

(c) The projectile strikes the ground when $y(t) = 0$:

$$\begin{aligned} 20\sqrt{2}t - 4.9t^2 &= 0 \\ t(20\sqrt{2} - 4.9t) &= 0 \\ \implies t &= 0, \frac{20\sqrt{2}}{4.9}. \end{aligned}$$ *

We use the later time:

$$t = \boxed{\frac{20\sqrt{2}}{4.9}}$$ *

The horizontal distance traveled, in meters, is

$$x\left(\frac{20\sqrt{2}}{4.9}\right) = 20\sqrt{2}\left(\frac{20\sqrt{2}}{4.9}\right) \quad *$$

$$= \boxed{\frac{800}{4.9}} \quad **$$

(d) Differentiating the equations in part (a), we attain

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 20\sqrt{2}\,, \quad *$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 20\sqrt{2} - 9.8t\,. \quad *$$

The flight duration (in seconds) is $20\sqrt{2}/4.9$, so the arc length is

$$L = \int_0^{20\sqrt{2}/4.9} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \quad *$$

$$= \boxed{\int_0^{20\sqrt{2}/4.9} \sqrt{(20\sqrt{2})^2 + (20\sqrt{2} - 9.8t)^2}\,\mathrm{d}t} \quad *$$